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SEMIPRIME ANTI FLEXIBLE RINGS

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ABSTRACT

In this paper we prove that the commutator is in the middle nucleus of an anti flexible ring and an anti flexible ring without non-zero nilpotent elements is flexible. Using these properties to show that a semi prime anti flexible ring is weakly standard ring. At the end of this paper we give an example of an anti flexible ring which is not a weakly standard ring.

KEYWORDS: Commutator, Nucleus, Anti Flexible Ring, Flexible Ring, Semi Prime, Weakly Standard Ring

INTRODUCTION

Anti flexible rings were introduced by Koiser [3] and a subclass of anti flexible rings was studied by Klienfled [2]. Simple anti flexible rings are characterized by Anderson and Outcalt [1]. In this paper, we discuss some properties of an anti flexible ring R. In general an anti flexible ring is not a weakly standard ring. We prove that the commutator is in the middle nucleus and an anti flexible ring without non-zero nilpotent elements is flexible. We use these properties to show that R is a weakly standard ring. Also we present an example of an anti flexible ring which is not a weakly standard ring.

PRELIMINARIES

A weakly standard ring R, where the associator (x, y, z) = (xy) z – x (yz) and the commutator (x, y) = xy – yx. The nucleus N of R is defined as the set of all elements n in R such that (n, R, R) = (R, n, R) = (R, R, n) = 0. We define a ring R is semi prime if for any ideal A of R, $A^2 = 0$ implies A = 0.We know that the ring R is said to be anti flexible if

A(x, y, z) = (x, y, z) – (z, y, x) = 0 (1)

Throughout this paper we assume that R is an anti flexible ring of

Characteristic ≠ 2 and that (x, x, x) = 0 (2)

Is an identity in R. Using the identity (1) in the linearization of (2), we obtain the identity

B(x, y, z) = (x, y, z) + (y, z, x) + (z, x, y) = 0 (3)

We shall also require the Teichmuller identity (which holds in any ring):

C(w, x, y, z) = (wx, y, z) – (w, xy, z) + (w, x, yz) – w(x, y, z) – (w, x, y) z = 0 (4)

In any ring (xy, z) = x(y, z) + (x, z) y +(x, y, z) + (z, x, y) – (x, z, y), from which we subtract

A(x, z, y) + B(x, y, z) = 0 to obtain

D(x, y, z) = (xy, z) – x(y, z) – (x, z) y + 2(x, z, y) = 0 (5)

Expanding $0 = C(w, x, y, z) – C(z, y, x, w)$, and using

$0 = A(z, y, xw) = A(z, yx, w) = A(zy, x, w) = ZA(y, x, w) = A(z, y, x) w$,

We get

$$0 = E(w, x, y, z) = ((w, x), y, z) – (w, (x, y), z + (w, x, (y, z)) - (w, (x, y, z)) - ((w, x, y), z). \quad (6)$$

Then we expand

$0 = E(w, x, y, z) + E(x, y, z, w) + E(y, z, w, x) + E (z, w, x, y)$

$-B((w, x), y, z) – B ((x, y), z, w) – B ((y, z), w, x) – B ((z, w), x, y)$ to get

$$0 = F(w, x, y, z) = (w, (x, y), z) + (x, (y, z), w) + (y, (z, w), x) + (z, (w, x), y) \quad (7)$$

Now we are able to derive the important identity $(w, (x, y), z) = 0$ (8)

Using these identities first we prove the following lemmas.

MAIN RESULTS

Lemma 1: *In an anti flexible ring R, commutators are in the middle nucleus.*

Proof: By expanding

$0 = E(x, x, y, x) + E(y, x, x, x)-B(x, x, (y, x)) + (B(x, x, y), x)$, we get

$0 = (x, (x, y, x))$, hence from $0 = (x, B(x, y, x))$ and $0 = (x, A(x, x, y))$, we have

$$0 = (x, (x, y, x)) = (x, (x, x, y)) = (x, (y, x, x,)) \quad (9)$$

Then it follows from (9) and $0 = E (y, x, x, x)$ that $((y, x), x, x) = 0$. Hence $(x, x,(y, x)) = 0$ by (1) and then from $0 = B(x, x,(y, x))$, we have

$$0 = (x, (y, x), x) \quad (10)$$

Substituting $x + z$ for x in (10) and subtracting

$0 = A (z, (y, x), x) + A (z, (y, z), x)$, we obtain

$$2(x, (y, x), z) + 2(x, (y, z), z) + (x, (y, z), x) + (z, (y, x), z) = 0 \quad (11)$$

Substituting $–z$ for z in (11) and then adding to (11) yields

$$2(x, (y, z), z) + (z, (y, x), z) = 0 \quad (12)$$

Next we linearize (12) and add $A (w, (y, x), z) = 0$ to get

$G(w, x, y, z) = (x, (y, z), w) + (x, (y, w), z) + (w, (y, x), z) = 0.$

Computing

$0 = F (w, x, y, z) + G (w, x, y, z) + G(x, y, w, z) – A (z, (w, x), y)$, we get

$H(w, x, y, z) = (x, (y, z), w) + (y, (w, x), z) = 0.$

Now, identity (8) follows from the expansion of

$0 = H(w, x, y, z) + H(x, w, y, z) - A(w, (y, z), x)\ 0$

Lemma 2: *In an anti flexible ring, $(x, y, x)^2 = 0$.*

Proof: By forming $0 = C(w, x, y, z) - C(x, y, z, w) + C(y, z, w, x) - C(z, w, x, y)$, and using (3),

We obtain

$$I(w, x, y, z) = (w, (x, y, z)) - (x, (y, z, w)) + (y, (z, w, x)) - (z, (w, x, y)) = 0 \quad (13)$$

If in (9) we replace x by both x + z and –x + z, adds and divides by 2, we obtain

$(z, (x, y, x)) + (x, (z, y, x)) + (x, (x, y, z)) = 0,$

$$(z, (x, x, y)) + (x, (z, x, y)) + (x, (x, z, y)) = 0 \quad (14)$$

And $(z, (y, x, x)) + (x, (y, z, x)) + (x, (y, x, z)) = 0.$

Because of (14) we have

$0 = I(x, y, x, z)$

$= (x, (y, x, z)) - (y, (x, z, x) + (x, (z, x, y)) - (z, (x, y, x))$

$= (x, (y, x, z)) - (y, (x, z, x)) - (z, (x, x, y)) - (x, (x, z, y)) + (x, (z, y, x)) + (x, (x, y, z))$

$= 2(x, (x, y, z)) + (x, (y, x, z) + (x, (x, z, y))$

$= (x, (x, y, z) + (y, z, x) + (z, x, y)) + (x, (x, y, z)) = 0.$

$$\text{Consequently} \quad (x, (x, y, z)) = 0 \quad (15)$$

By linearizing (15) we obtain

$$J(w, x, y, z) = (w, (x, y, z)) + (x, (w, y, z)) = 0 \quad (16)$$

We define u by $u = (x, y, x)$. Then $0 = J(xy, x, y, x)$ implies $-(xy, u) = (x, (xy, y, x))$.

If we take $z = xy$ in (15), $(x, (x, y, xy)) = 0$ implies $(x, (xy, y, x)) = 0$, using (1). Therefore $(xy, u) = 0$. Because of (9) we have $(u, x) = 0$ and $((y, x, x), y) = 0$. But then $(B(y, x, x) + A(y, x, x) -$

$2(y, x, x), y) = 0$ implies $(u, y) = 0$. From (5), by taking $z = u$, we get $(x, u, y) = 0$.

Using (3) we obtain $(x, y, u) = -(u, x, y) = (y, u, x) = 0$.

$$\text{Thus } (u, x, y) = 0 \quad (17)$$

If we put $x = u$, $y = x$, $z = x$ in (5) we get $(ux, x) = 0$.

By linearizing this, $(ux, y) + (uy, x) = 0$.

Then (5) implies that $((ux)y, x) = ux(y, x) + 2(ux, y, x)$.

By interchanging x and y in this equation, we have $((ux)x, y) = ux(x, y) + 2(ux, x, y)$.

By adding these two equations, we obtain

$$(ux, y, x) + (ux, x, y) = 0 \quad (18)$$

From (3), (ux, y, x) + (y, x, ux) + (x, ux, y) = 0.

Then – (ux, x, y) + (y, x, ux) + (x, ux, y) = 0, using (18).

This becomes -(y, x, ux) + (y, x, ux) + (x, ux, y) = 0, using (1).

So (x, ux, y) = 0. Also (18) implies that (ux, y, x) = -(y, x, ux) = (x, ux, y) = 0. (19)

By using (19) and (17) in (4), we get that u(x, y, x) = 0 and hence $(x, y, x)^2 = 0$. 0

Theorem 1: If R is an anti flexible ring without non-zero nilpotent elements, then R is a weakly standard ring.

Proof: Since R is without nonzero nilpotent elements, lemma (2) implies that

(x, y, x) = 0 (20)

So R is flexible. Since R is flexible, we have (x, y, z) = - (z, y, x).

By taking z = v as a commutator, we obtain (x, y, v) = - (v, y, x). From (1), it follows that

2(x, y, v) = 0 and hence (x, y, v) = 0 (21)

Since R is of characteristic ≠ 2.Now from (20), lemma (1) and (21), we get that R is a

Weakly standard ring 0

Now we give an example of an anti flexible ring which is not a weakly standard ring.

Example 1: Suppose that the ring R is defined by the following multiplication table together with all finite sums of e, a, b, c, d, h such that x + x = 2x ≠ 0.

	e	a	b	c	d	h
e	e	b	a	0	0	0
a	h	c	0	0	0	0
b	0	0	0	0	0	0
c	0	0	0	0	0	0
d	0	0	0	0	0	0
h	h+b	b	0	0	0	0

We observe that (a, a, a) = a^2 a-a a^2 = ca – ac = 0.

Therefore, R satisfies (x, x, x) = 0.

This is enough to check the identity (x, y, z) = (z, y, x) .

Now (e, e, a) = ea-e (ea) = b and

(a, e, e) = (ae)e – ae = he – h = h + b – h = b.

Thus (e, e, a) = (a, e, e).

Next we see that the commutator is not in the left nucleus.

((e, b), e, e) = (eb, e, e) – (be, e, e)

= (a, e, e) = b ≠ 0.

Hence R is not a weakly standard ring. 0

CONCLUSIONS

In this paper we proved that the commutator is in the middle nucleus of an anti flexible ring and an anti flexible ring without non-zero nilpotent elements is flexible. Using these properties we proved that a semi prime anti flexible ring is a weakly standard ring.

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